

## BRIEF COMMUNICATION

# EVALUATION OF SEVERAL CORRELATIONS USED FOR THE PREDICTION OF PRESSURE DROP IN PARTICULATE FLOWS

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### 1. INTRODUCTION

In the past 30 years the subject of pressure drop in gas–solid systems has been examined by several experimentalists. Thus, several sets of experimental data have been compiled and about two dozen correlations are known to have emanated from these data. Rose & Duckworth (1969) have shown that the pressure drop in gas–solid pipelines is a function of at least 6 dimensionless groups. One can use as many as 12 dimensionless groups (Martin & Michaelides 1985) if one considers all the solid particle characteristics that could possibly influence the pressure drop in pipelines. Of these groups, some play an important role in the determination of pressure drop and are present in most correlations, others are less important and rarely appear.

The most frequently used correlations appear in handbooks, such as those by Hetsroni (1982) or Govier & Aziz (1977), and are recommended for certain ranges of their parameters. Many of the published correlations are difficult to use, either because they contain parameters which are difficult to measure or evaluate (e.g. particle sphericity or shape factors) or because they require the use of graphical functions, which are used to represent the contributions of all the dimensionless groups.

Few of the researchers have provided any indication as to the accuracy and limitations of their correlations. This creates a problem for designers who wish to use a correlation, but do not know the limits of its applicability as far as loading, particle sizes and pipe diameters are concerned. The problem is even more acute if one tries to decide which of the many available correlations to use in a particular situation. The present project was undertaken for this reason: to provide a critical evaluation of the many correlations which have appeared and the degree to which these correlations agree with several experimental data sets. Thus, several data sets were collected and arranged in a way that may be used to give all the parameters present in the correlations. Then each correlation was compared to each of the data sets.

During the literature search a special effort was made to collect the original data on which the authors based their correlations. This was possible only to a limited extent, because a lot of the data were not published with the correlation or because the original publication was not available; in other cases the expressions presented by one author were based on the experimental data of someone else. Nevertheless, the data sets of 14 different authors were obtained containing approx. 1450 experimental data between them. After examining the reports a few of these data sets were judged to be inappropriate to use for two reasons: either because inadequate allowance was made for the acceleration of solids in the pipe or because the variables listed were incomplete and, hence, the data sets could not be used with some correlations. Graphically presented data were usable with difficulty and in these cases a great deal of effort was spent interpreting the graphical information correctly.

### 2. DATA SETS USED FOR THE STUDY

Table 1 summarizes the sources used for gathering the experimental data for this study.

Table 1

Author(s)	Year	Loadings <sup>a</sup>	Substance
Degliobizzi <i>et al.</i>	1983	L, I, H	Plastics
Hariu & Molstad	1949	L, I	Sand, silicon alumina catalyst
Hinkle	1953	L, I	Plastics, aluminum
Hitchcock & Jones	1958	L, I	Peas, glass
Koncheski <i>et al.</i>	1975	L, I	Coal
Rose & Barnacle	1957	L, I	Mustard seed
Uematu & Morikawa	1960	L, I	Rape seed
Voigt & White	1948	L, I, H	Sand, glass
Welshof	1962	L, I	Grain

<sup>a</sup>L = low, I = intermediate, H = high.

All the data sets were fed into a computer. The following information was used with all the correlations examined:

- (a) the pipe diameter,  $D$ ;
- (b) the solids loading,  $m^*$ ;
- (c) the solids/air density ratio,  $\rho_s/\rho_a$ ;
- (d) the particle/pipe diameter ratio,  $d/D$ ;
- (e) the Reynolds number,  $Re = U_a \rho_a D / \mu_a$  (where  $U_a$  is the air superficial velocity);
- (f) the Froude number,  $Fr = U_a^2 / gD$ ;
- (g) the frictional pressure drop per unit length,  $\Delta P_f / \Delta L$ ;

and

- (h) the friction factor,  $f = (\Delta P_f / \Delta L) / (\rho_a U_a^2 / 2D)$ .

This information was adequate for the critical evaluation of all the correlations examined in this study. Wherever the friction factor for air flowing alone in the pipe was needed, the following expression was used:

$$f_a = 4(0.0014 + 0.125 Re^{-0.32}). \quad [1]$$

### 3. METHOD OF COMPARISON

The method used for evaluation was to find out how well a given correlation predicted the individual sets of data under the same conditions in which the experiments were conducted. First, all correlations were arranged to yield the D'Arcy friction factor  $f$ :

$$f = \frac{\frac{\Delta P_f}{\Delta L}}{\frac{1}{2D} \rho_a U_a^2}, \quad [2]$$

where  $\Delta P_f$  is the total frictional pressure drop for the solids and the gas flowing in the pipe and  $\Delta L$  is the length of pipe over which  $\Delta P_f$  is observed.

A relative deviation of the experimental data points and those predicted by the correlation was defined as

$$e_i = \frac{f_i - \hat{f}_i}{f_i}, \quad [3]$$

where  $f_i$  is the quantity observed in the experiment and  $\hat{f}_i$  the quantity calculated by the correlation under the same conditions in which the experiment took place.

From the relative deviation, an average relative deviation, an absolute deviation and a standard

deviation of the error were obtained as follows:

$$\bar{e} = \frac{1}{N} \sum_1^N e_i, \quad [4]$$

$$|\bar{e}| = \frac{1}{N} \sum_1^N |e_i| \quad [5]$$

and

$$\sigma = \frac{1}{N-1} \sum_1^N (e_i - \bar{e})^2, \quad [6]$$

where  $N$  is the number of data points in a set.

The standard deviation is mostly biased by the very high relative deviations (because of the squaring operation), while  $|\bar{e}|$  treats equally high or low relative deviations. For this reason the present authors place more importance on a low value of  $|\bar{e}|$  than of  $\sigma$ .

Table 2 lists  $\bar{e}$ ,  $|\bar{e}|$  and  $\sigma$  for all the correlations used, as evaluated for all the sets of data at hand.

#### 4. THE CORRELATIONS

Two basic statistical models have been used in the past for the generation of pressure-drop correlations: model A assumes that the air and solid contributions are additive and model B assumes that the total friction factor is a multiple of the air friction factor  $f_a$ . The functional forms of the two models are

$$f = f_a + \alpha f_s \quad [7]$$

and

$$f = f_a (\beta + \phi)^n, \quad [8]$$

where  $f_a$ ,  $f_s$ ,  $\alpha$ ,  $\beta$ ,  $\phi$  and  $n$  may be constants or functions of the air and solid flow variables.

The correlations used in this study are based on both models A and B, as the reader may easily observe. A list of these correlations is given below.

##### 4.1. Barth (1958)

$$f = f_a + 0.005 m^* \frac{1 - \frac{1}{Fr}}{1 + 0.00125 Fr_0^2}, \quad [9]$$

where  $Fr_0$  is based on the settling velocity of particles.

##### 4.2. Belden & Kassel (1949)

$$f = (1 + m^*) \left[ 0.049 + 0.22 \frac{m^*}{(1 + m^*)^2} \right] Re^{-0.2}. \quad [10]$$

##### 4.3. Dogin & Lebedev (1962)

$$f = f_a + Cm^* \left( \frac{d}{D} \right)^{0.1} Re^{0.4} Fr^{-0.5} \left( \frac{\rho_s}{\rho_a} \right), \quad [11]$$

where the constant  $C$  has the value  $6.6 \times 10^{-6}$ , as suggested by the authors, or  $8 \times 10^{-7}$ , as suggested by Soo in Hetsroni (1982). This was treated as two expressions; D&L1, based on the first value of  $C$ ; and D&L2, based on the second value.

##### 4.4. Hinkle (1953)

$$f = f_a + m^* \left( \frac{U_p}{U_a} \right)^2 f_p, \quad [12]$$

Table 2

Correlation	Data file	Hariu & Molstad (1949)	Hinkie (1953)	Hiechock & Jones (1958)	Koncheski <i>et al.</i> (1975)	Rose & Barnacle (1957)	Uematu & Morikawa (1960)	Voigt & White (1948)	Weishof (1962)	Degliobizzi <i>et al.</i> (1983)
Rose & Duckworth (1969)	$ \bar{r} $	0.502	0.175	0.446	0.295	0.138	0.254	0.417	0.256	0.383
	$\bar{r}$	0.502	0.163	0.446	0.258	0.077	0.228	0.308	0.248	0.383
	$\sigma$	0.163	0.125	0.107	0.236	0.147	0.164	0.338	0.236	0.142
Rose & Barnacle (1957)	$ \bar{r} $	0.277	0.243	0.525	0.395	0.195	0.274	0.354	0.331	0.515
	$\bar{r}$	0.265	0.236	0.525	0.393	0.139	0.248	0.233	0.329	0.515
	$\sigma$	0.146	0.140	0.099	0.209	0.166	0.172	0.313	0.254	0.198
Dogin & Lebedev (1962), D&L1	$ \bar{r} $	0.344	0.502	2.881	5.374	0.732	0.074	0.521	2.216	1.311
	$\bar{r}$	0.173	0.487	-2.881	-5.374	-0.71	-0.062	-0.496	-2.216	-1.224
	$\sigma$	0.445	0.463	1.216	2.784	0.571	0.087	0.383	1.253	1.039
Dogin & Lebedev (1962), D&L2	$ \bar{r} $	0.489	0.179	0.194	0.368	0.139	0.257	0.373	0.103	0.323
	$\bar{r}$	0.489	0.165	-0.155	-0.267	0.076	0.231	0.255	0.064	0.323
	$\sigma$	0.154	0.137	0.156	0.403	0.142	0.164	0.319	0.134	0.124
Pfeffer <i>et al.</i> (1966), PLR1	$ \bar{r} $	0.264	0.106	0.313	0.246	0.185	0.126	0.297	0.190	0.351
	$\bar{r}$	0.262	0.037	-0.313	0.119	-0.16	0.094	-0.015	0.056	0.351
	$\sigma$	0.162	0.145	0.119	0.270	0.17	0.115	0.355	0.236	0.148
Barth (1958)	$ \bar{r} $	0.246	0.092	0.191	0.264	0.314	0.15	0.322	0.194	0.166
	$\bar{r}$	0.246	0.013	-0.188	-0.036	-0.291	0.123	-0.183	-0.154	0.141
	$\sigma$	0.098	0.144	0.132	0.359	0.258	0.121	0.345	0.176	0.156
Richardson & McLeman (1960)	$ \bar{r} $	1.96	0.165	0.718	1.7	0.121	0.184	0.419	0.288	0.672
	$\bar{r}$	1.261	0.130	-0.104	-1.7	0.057	0.152	0.309	-0.288	-0.577
	$\sigma$	4.586	0.143	1.552	0.932	0.136	0.144	0.339	0.14	0.682

Hinkle (1953)	$ \bar{r} $	0.428	0.444	0.287	0.344	0.137	0.266	0.28	0.282	2.31
	$\bar{r}$	0.417	0.197	0.257	0.272	0.071	0.239	0.132	0.278	-2.156
	$\sigma$	0.216	0.659	0.232	0.308	0.154	0.17	0.313	0.228	6.096
Belden & Kassel (1949)	$ \bar{r} $	0.188	0.18	0.161	0.356	0.655	0.226	0.606	0.422	0.272
	$\bar{r}$	0.099	0.016	-0.001	-0.256	-0.363	0.226	-0.452	-0.369	0.231
	$\sigma$	0.21	0.231	0.192	0.409	0.640	0.227	0.627	0.358	0.309
Shimizu <i>et al.</i> (1978)	$ \bar{r} $	0.537	0.19	0.191	0.449	0.721	0.071	0.91	0.65	0.168
	$\bar{r}$	0.534	0.156	-0.113	-0.396	-0.699	-0.058	-0.902	-0.644	0.032
	$\sigma$	0.382	0.198	0.233	0.457	0.534	0.08	0.728	0.403	0.197
Pfeffer <i>et al.</i> (1966), PLR2	$ \bar{r} $	0.35	0.163	0.379	0.285	0.149	0.146	0.357	0.115	0.419
	$\bar{r}$	0.31	0.146	-0.379	0.253	-0.123	0.119	-0.246	0.083	0.419
	$\sigma$	0.364	0.132	0.010	0.217	0.168	0.116	0.380	0.141	0.148
Pfeffer <i>et al.</i> (1966), PLR3	$ \bar{r} $	0.26	0.435	0.452	0.426	0.348	0.349	0.367	0.262	0.651
	$\bar{r}$	0.24	0.430	-0.452	0.419	0.149	0.349	-0.056	0.262	0.651
	$\sigma$	0.206	0.129	0.106	0.189	0.466	0.285	0.47	0.14	0.178
Michaelides (1987)	$ \bar{r} $	0.367	0.168	0.337	0.22	0.091	0.194	0.326	0.08	0.241
	$\bar{r}$	0.367	0.151	-0.337	-0.037	-0.001	0.168	0.198	-0.004	0.241
	$\sigma$	0.147	0.132	0.116	0.269	0.133	0.135	0.31	0.11	0.101
Hitchcock & Jones (1958)	$ \bar{r} $	0.4	0.169	0.229	0.276	0.162	0.243	0.364	0.233	0.993
	$\bar{r}$	0.375	0.1	-0.165	0.075	0.103	0.216	0.253	0.219	-0.902
	$\sigma$	0.249	0.178	0.317	0.347	0.149	0.158	0.313	0.214	2.41
Koncheski <i>et al.</i> (1975)	$ \bar{r} $	1.46 · 10 <sup>10</sup>	0.578	0.369	0.22	3.027	8.322	5.1 · 10 <sup>3</sup>	0.237	0.607
	$\bar{r}$	1.46 · 10 <sup>10</sup>	0.573	-0.092	-0.077	-2.627	-8.019	-5.1 · 10 <sup>3</sup>	0.091	0.607
	$\sigma$	1.49 · 10 <sup>10</sup>	0.183	0.440	0.281	2.715	5.251	3.68 · 10 <sup>3</sup>	0.291	0.229

where  $U_p$  is the solids velocity and  $f_p$  is a function of the solid and air properties.

#### 4.5. The Hitchcock and Jones correlation (1958)

$$f = f_a + 0.003 m^{*0.9} Fr^{-0.5} \left(\frac{d}{D}\right)^{-0.9} \quad [13]$$

#### 4.6. Koncheski et al. (1975)

$$\frac{\Delta P_f}{\Delta L} = 0.00454 \dot{m}_s^{0.688} \gamma^{0.410} \exp\left(\frac{7.833}{D}\right) \quad [14]$$

This is the only correlation not expressed in dimensionless form. It is valid in its present form in the British system of units and  $\dot{m}_s$  is the solids mass flow rate,  $\gamma$  is the specific gravity of solids and  $D$  must be in inches.

#### 4.7. The expression derived by Michaelides (1987)

$$f = f_a + 0.076 \frac{m^*}{\sqrt{Fr}} \quad [15]$$

This is derived from the sets of all the available data and naturally appears to be the best of the group examined in this study.

#### 4.8. The study by Pfeffer et al. (1966)

This work is a source of experimental correlations for frictional pressure drop and for heat-transfer coefficients. In the study it is recommended that the following expression be used for  $f$  (PLR1):

$$f = f_a(1 + m^*)^{0.3} \quad [16]$$

In the present study it was found that [16] in general underpredicts the data, a fact corroborated by Soo (Hetsroni 1982).

Two more expressions were found in the Pfeffer et al. study, both of which merit consideration. They both emanate from heat-transfer equations after Reynolds analogy has been applied and they are representative of models A and B, as discussed at the beginning of this section:

$$f = f_a(1 + 4 Re^{-0.32} m^*), \quad [17]$$

labeled PLR2 here; and

$$f = 7.6 f_a m^{*0.45} Re^{-0.21}, \quad [18]$$

labeled PLR3.

#### 4.9. Richardson & McLeman (1960)

These authors suggest the expression

$$f = f_a \left(1 + \frac{45000}{V_0 U_p^2} \dot{m}_s\right), \quad [19]$$

where  $V_0$  is the settling velocity of the particles and  $U_p$  the actual velocity of the particles both in ft/s;  $\dot{m}_s$  must be in lb/s.

#### 4.10. Rose & Barnacle (1957)

$$f = f_a + \frac{\pi}{2} m^* \left(\frac{\rho_s}{\rho_a}\right)^{0.5} \psi, \quad [20]$$

where  $\psi$  is a function of the Reynolds number of the flow and is given graphically in the original study. This function was represented by spline polynomials in the computer program used for the present work.

4.11. *Rose & Duckworth (1969)*

These authors suggested a correlation of the following form:

$$f = f_a + \phi_1(m^*)\phi_2\left(\frac{d}{D}\right)\phi_3(\text{Fr})\phi_4\left(\frac{\rho_s}{\rho_a}\right), \quad [21]$$

where  $\phi_i(x)$  are functions of the variable  $x$  in the parentheses.

4.12. *Shimizu et al. (1978)*

$$f = f_a(1 + 0.379 m^*). \quad [22]$$

This is suggested as an approximate expression for the friction factor.

It must be pointed out here that one may find in the literature other data correlations, such as those by Muschelknautz (1959), Mason & Boothroyd (1971), Wirth & Molerus (1983), Schuchart

Table 3

Correlations	Data file	All data
Rose & Duckworth (1969)	$ \bar{e} $	0.324
	$\bar{e}$	0.284
	$\sigma$	0.252
Rose & Barnacle (1957)	$ \bar{e} $	0.368
	$\bar{e}$	0.347
	$\sigma$	0.24
Dogin & Lebedev (1962), D&L1	$ \bar{e} $	2.942
	$\bar{e}$	-2.904
	$\sigma$	3.079
Dogin & Lebedev (1962), D&L2	$ \bar{e} $	0.343
	$\bar{e}$	0.021
	$\sigma$	0.435
Pfeffer <i>et al.</i> (1966), PLR1	$ \bar{e} $	0.255
	$\bar{e}$	0.127
	$\sigma$	0.286
Barth (1958)	$ \bar{e} $	0.245
	$\bar{e}$	-0.013
	$\sigma$	0.336
Richardson & McLeman (1960)	$ \bar{e} $	1.132
	$\bar{e}$	-0.938
	$\sigma$	1.857
Hinkle (1953)	$ \bar{e} $	0.647
	$\bar{e}$	-0.151
	$\sigma$	2.577
Belden & Kassel (1949)	$\bar{e}$	-0.175
	$\sigma$	0.478
Shimizu <i>et al.</i> (1978)	$ \bar{e} $	0.467
	$\bar{e}$	-0.407
	$\sigma$	0.526
Pfeffer <i>et al.</i> (1966), PLR2	$ \bar{e} $	0.292
	$\bar{e}$	0.125
	$\sigma$	0.341
Pfeffer <i>et al.</i> (1966), PLR3	$ \bar{e} $	0.419
	$\bar{e}$	0.307
	$\sigma$	0.366
Michaelides (1987)	$ \bar{e} $	0.236
	$\bar{e}$	0.087
	$\sigma$	0.276
Hitchcock & Jones (1958)	$ \bar{e} $	0.395
	$\bar{e}$	-0.021
	$\sigma$	1.066
Koncheski <i>et al.</i> (1975)	$ \bar{e} $	$0.167 \cdot 10^{10}$
	$\bar{e}$	$-0.167 \cdot 10^{10}$
	$\sigma$	$0.685 \cdot 10^{10}$

(1970) among others. These correlations were excluded from the present study for one or more of the following reasons:

- (a) They required knowledge of flow parameters other than those contained in the data sets.
- (b) They pertain to a very specific flow regime (e.g. slug flow, dense phase etc.).
- (c) The information provided was of a graphical nature which could not be represented easily by spline polynomials.
- (d) Their validity was limited to special flows or systems.

After comparing each correlation with all the data sets individually, the data from all the sources were combined in a single set and the correlations compared with this large data bank. The results are given in table 3, where  $\bar{e}$ ,  $|\bar{e}|$  and  $\sigma$  are given for each correlation. Figure 1 shows these results in graphical form for 11 selected correlations with the minimum fractional errors.

At this point it must be emphasized that a good correlation is characterized by a value of  $\bar{e}$  which is close to zero (no bias towards overpredicting or underpredicting) and low  $|\bar{e}|$ , which signifies that

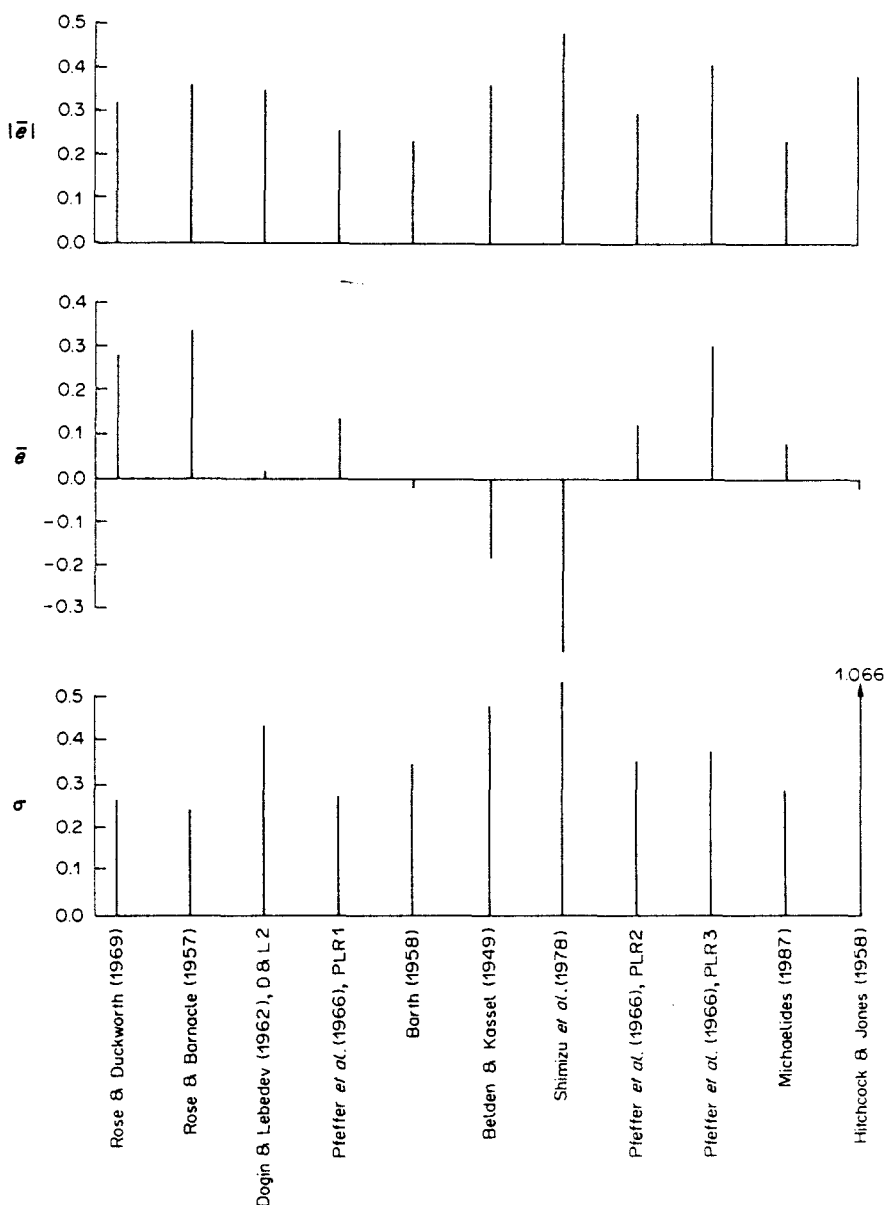


Figure 1. Graphical representation of the results.



the absolute errors are not large. A low value of  $\sigma$  and  $|\bar{\epsilon}|$  will ensure that the spread of the deviations from their mean value is not high and this may account for the consistency of a correlation.

## 5. CONCLUSIONS

It is apparent that the expression proposed by Michaelides (1987) is the best to correlate the sets of data used in the present study. This should not be surprising, given that the expression emanates from these data sets. It is suggested, however, that for better results the value of the constant (0.076) is taken from the original paper.

The expressions suggested by Barth (1958) and Rose & Duckworth (1969) appear to be acceptable for the prediction of the pressure drop in general. Pfeffer *et al.* (1966) (PRL1), in general, underpredict the data, as corroborated by Soo in Hetsroni (1982). Other correlations may be acceptable with some types of solids.

Of the correlations examined, that of Koncheski *et al.* (1975) should not be used at all for pipe diameters < 50 mm. The original expression suggested by Dogin & Lebedev (1962) overpredicts consistently and the modified one suggested by Soo in Hetsroni (1982), is at least a better expression than the original.

All other expressions examined are in the middle category and one may draw conclusions about their applicability by consulting tables 2 and 3 of the present work.

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